GCSE Maths – Geometry and Measures

Vector Operations

Worksheet

NOTES

SOLUTIONS

This worksheet will show you how to work out different types of vector operation questions. Each section contains a worked example, a question with hints and then questions for you to work through on your own.

Section A

Worked Example

Find $a + b$ **when** $a = \begin{pmatrix} 5 \\ -2 \end{pmatrix}$, $b = \begin{pmatrix} -3 \\ 0 \end{pmatrix}$.

Step 1: Write $a + b$ as one column vector by adding each of the rows.

$$a + b = \begin{pmatrix} 5 \\ -2 \end{pmatrix} + \begin{pmatrix} -3 \\ 0 \end{pmatrix} = \begin{pmatrix} 5 + (-3) \\ -2 + 0 \end{pmatrix}$$

Step 2: Sum each row and calculate the total.

$$5 + -3 = 2$$

$$-2 + 0 = -2$$

$$a + b = \begin{pmatrix} 2 \\ -2 \end{pmatrix}$$

Guided Example

Let $a = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$ **and** $b = \begin{pmatrix} 7 \\ 1 \end{pmatrix}$. **Write** $2a + b$ **as a column vector.**

Step 1: Multiply both components of a by 2.

Step 2: Add the components of $2a$ to the components of b.

Now it's your turn!
If you get stuck, look back at the worked and guided examples.

1. Three vectors are listed below with some missing values

$$a = \begin{pmatrix} 3 \\ 2 \end{pmatrix} \quad b = \begin{pmatrix} d \\ e \end{pmatrix} \quad c = \begin{pmatrix} 1 \\ f \end{pmatrix}$$

Use the following calculations to find the value of d, e and f:

$$a + b = \begin{pmatrix} 3 \\ 0 \end{pmatrix}$$

$$2c + b = \begin{pmatrix} 2 \\ 2 \end{pmatrix}$$

2. Let $a = \begin{pmatrix} 1 \\ 6 \end{pmatrix}$ and $b = \begin{pmatrix} 3 \\ -2 \end{pmatrix}$.

Write the following as column vectors:

a) 2**a** – 3**b**

b) **a** – **b**

c) 2**b** – 3**a**

d) 4**a** + 2**b**

Section B

Worked Example

What is vector b **if** $b = 3a$ **and** $a = \begin{pmatrix} -2 \\ 5 \end{pmatrix}$**?**

Step 1: Multiply each row of vector a by the scalar 3.

$$3a = 3\begin{pmatrix} -2 \\ 5 \end{pmatrix} = \begin{pmatrix} 3(-2) \\ 3(5) \end{pmatrix}$$

Step 2: Calculate each new value and write the new vector b.

$$3 \times -2 = -6$$
$$3 \times 5 = 15$$

$$b = \begin{pmatrix} -6 \\ 15 \end{pmatrix}$$

Guided Example

Let $a = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$ **and** $b = \begin{pmatrix} -2 \\ 7 \end{pmatrix}$ **and** $c = \begin{pmatrix} -1 \\ 3 \end{pmatrix}$.

Write $a - 2b + c$ **as a column vector.**

Step 1: Calculate $2b$ by multiplying each component of b by 2.

Step 2: Calculate $a - 2b$.

Step 3: Add the vector **c** to $a - 2b$ to find $a - 2b + c$.

Now it's your turn!
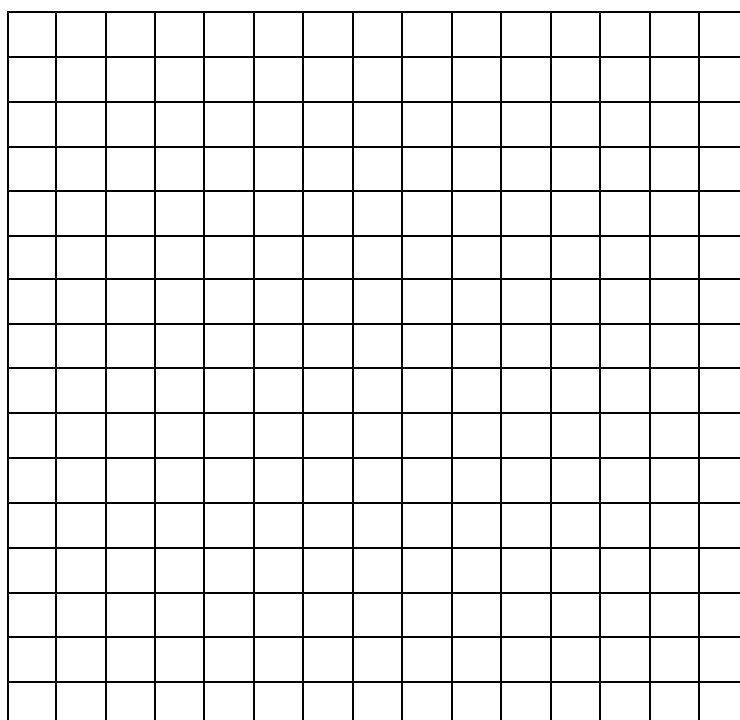If you get stuck, look back at the worked and guided examples.

3. Given the vectors $a = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$ $b = \begin{pmatrix} 1 \\ 5 \end{pmatrix}$, draw and label the following vectors from the origin on the axes below:

   a) **a**

   b) **a + b**

   c) 3**a**

   d) 3**b** − 2**a**

4. Given the vectors $a = \begin{pmatrix} 1 \\ 4 \end{pmatrix}$ $b = \begin{pmatrix} -3 \\ 6 \end{pmatrix}$ $c = \begin{pmatrix} -1 \\ -2 \end{pmatrix}$, write the following expressions as a single column vector:

a) **a** + **b**

b) 2**c** + **b**

c) 3**a** − 2**c**

d) **a** + **b** − 1.4**c**

e) 4**a** − **b** + 3**c**